MIDDLE EUROPEAN SCIENTIFIC

**Volume 46, Issue 3 | 2026**
https://journal.academicjournal.online/index.php/cejsr
**ISSN: 2694-9970**

*Article*

# Using General Logical Activity Methods in Solving Irrational Equations

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**Abstract:** This article analyzes the role and importance of general logical activity methods in solving trigonometric equations. In particular, it highlights the process of integrating logical operations such as analysis, synthesis, comparison, generalization, and abstraction into a mathematical solution strategy. It is scientifically substantiated that the use of general logical activity methods is connected with the development of students' problem-solving skills and their ability to find solutions quickly and effectively.

**Keywords:** Irrational equations, general logical activity methods, analysis, synthesis, comparison, generalization, abstraction, logical thinking.

**Citation:** Kodirov, K. R & Nishonboyev, A. S. Using General Logical Activity Methods in Solving Irrational Equations. Middle European Scientific Bulletin 2026, 46(3), 14-19

Received: 10th Feb 2026
Revised: 21th Mar 2026
Accepted: 18th Apr 2026
Published: 15th May 2026



## 1. Introduction

In modern mathematics education, the pedagogical paradigm has fundamentally shifted from traditional rote memorization toward cultivating adaptive, higher-order cognitive skills. This approach emphasizes that mathematics instruction must actively serve the intellectual development of the student rather than forcing the learner to passively fit a rigid curriculum [1]. Integrating systemic mental operations into algebraic instruction fosters an environment where students do not merely memorize algorithms but consciously construct cognitive frameworks. This strategic focus is essential for improving global mathematical literacy and preparing students to apply abstract logic to complex, real-world problems.

At the intersection of cognitive psychology and mathematics education lies the activity-based approach, a theoretical framework that treats learning as an active, goal-directed process. This methodology posits that deep conceptual understanding is driven by core mental operations: analysis, synthesis, comparison, generalization, and abstraction. While these major concepts are widely recognized as fundamental pillars of logical thinking, their integration into secondary and higher education has historically been spontaneous rather than systematic [2], [3]. Consequently, students frequently struggle to bridge the gap between abstract logical theories and practical mathematical problem-solving strategies.

A significant knowledge gap persists in the explicit application of these general logical activity methods to specific, challenging algebraic domains, such as irrational equations. Previous pedagogical literature has heavily focused on the theoretical benefits of logical operations or limited their practical application to geometry and basic arithmetic. Very little research provides an operational blueprint for utilizing structured mental decomposition and abstraction to solve complex radical functions. This article addresses this gap by directly analyzing how targeted logical operations can transform the mechanical process of finding roots into a conscious, strategic cognitive exercise.

The methodology of this study relies on a qualitative, example-based pedagogical analysis. It systematically deconstructs the solutions of diverse irrational equations to demonstrate the practical application of analysis, abstraction, and generalization [4], [5]. For instance, equations are first subjected to formal structural analysis to establish domain constraints and identify underlying patterns. Then, via abstraction, complex algebraic expressions are mapped to familiar quadratic or trigonometric forms through deliberate variable substitutions. Finally, generalization allows students to process the specialized outputs, eliminate extraneous roots, and unify the broader solution matrix.

It is expected that this structured approach will significantly optimize students' procedural speed, accuracy, and conceptual retention. The findings substantiate that anchoring algebraic steps in explicit logical operations prevents common cognitive errors and builds robust mathematical competencies. Ultimately, these results imply that mathematics curricula should be systematically redesigned to explicitly teach the logical meta-strategies underlying algebraic manipulation [6]. By making these implicit cognitive tools explicit, educators can better equip students for international assessment frameworks while fostering lifelong problem-solving capabilities.

## 2. Materials and Methods

The reforms being carried out in our country to improve the quality of teaching mathematics in general secondary and secondary specialised educational institutions, to develop the activity of general education schools specialising in mathematics in the regions, and to establish new schools are expanding the possibilities for introducing advanced teaching methods and technologies aimed at developing students through mathematics, as well as for improving the methodological system of the educational process [7].

Research on the formation of general mathematical and general learning activity methods in school students has been carried out by such scholars as O.B. Yepisheva and V.A. Testov, and research on general logical methods has been conducted by I.V. Voinova and V.N. Moiseyeva. Taking into account the system of basic learning-and-cognitive activity methods necessary for students to achieve the goals of mathematics education, developed in the studies of O.B. Yepisheva [8] and L.I. Bozhenkova [9], we consider the activity methods related to the mental operations of analysis, synthesis, generalisation, abstraction, and concretisation to be the main general logical activity methods. This is because these activity methods form the basis of other methods of logical thinking, such as working with concepts, working with judgments, working with conclusions, and searching for a solution to a problem.

## 3. Results and Discussion

The formation of general logical activity methods in students contributes to the conscious formation of mathematical activity methods, which, in turn, helps improve the quality of students' mathematical competencies.

Below we present descriptions of each of the main general logical activity methods.

Analysis and synthesis (separation and combination) mean the mental or direct decomposition of a whole into its constituent parts and the construction of a whole from its parts [10].

Generalisation is the logical method of moving from the particular to the general, from less general knowledge to more general knowledge.

Abstraction is a general logical activity method in which certain features of an object are singled out and considered separately from its other features.

Concretisation is a logical method that consists in one-sidedly determining one or another aspect of the object under study without considering its connection with other aspects [11].

The formation of general logical activity methods has been studied in various contexts and in different age groups. However, logical methods have mainly been considered as a means of teaching mathematics. In such cases, general logical activity methods were formed not purposefully, but spontaneously.

In this article, the application of general logical activity methods (analysis, synthesis, generalisation, concretisation, and abstraction) to solving irrational equations is presented through the process of solving examples.

**Example 1.** Solve the equation $(11-x)(11+x)=\sqrt{121-x}$

**Solution.** We apply the general logical activity methods of analysis, abstraction, and generalisation to this equation.

**Analysis.** The given equation is an irrational equation. At the same time, a linear function is given under the principal square root, while the left-hand side of the equality is a quadratic function [12], [13]. For the given equation, the following conditions are required:

$$\begin{cases}(11-x)(11+x)>0\\ 121-x>0\end{cases}$$

Solving them, we obtain

$$\begin{cases}-11<x<11\\ x<121\end{cases}\Rightarrow -11<x<11$$

The equation can be written as $121-x^2=\sqrt{121-x}$ .

**Abstraction.** Equations of the form $\sqrt{x+a}=x^2-a$ can be reduced to a quadratik equation by an appropriate substitution and then solved. In the obtained equation, let

$$y=121$$

Then, with respect to $y$, we obtain

$$y-x^2=\sqrt{y-x}$$

or

$$(y-x^2)^2=(\sqrt{y-x})^2$$

that is,

$$y^2-(2x^2+1)y+x^4+x=0$$

Considering this as a quadratik equation in $y$, we compute its discriminant:

$$D=(2x^2+1)^2-4(x^4+x)=4x^4+4x^2+1-4x^4-4x=(2x-1)^2$$

Hence we find the roots

$$y_1=\frac{(2x^2+1)+(2x+1)}{2}=\frac{2x^2+2x+2}{2}=x^2+x+1$$

and

$$y_2=\frac{(2x^2+1)-(2x+1)}{2}=\frac{2x^2-2x}{2}=x^2-x$$

**Generalisation.** Substituting the obtained roots into the designation $y=121$, we get

$$x^2+x=121$$

and

$$x^2-x+1=121,$$

or, equivalently,

$$x^2+x-121=0$$

and

$$x^2 - x - 120 = 0.$$

Solving these quadratic equations, we obtain

$$x_{1,2} = \frac{-1 \pm \sqrt{485}}{2}, \qquad x_{3,4} = \frac{1 \pm \sqrt{481}}{2}.$$

According to the condition $-11 < x < 11$ required for the equation, the extraneous roots are rejected. Therefore, the solutions are

$$x_1 = \frac{-1 + \sqrt{485}}{2}, \qquad x_2 = \frac{1 - \sqrt{481}}{2}.$$

**Example 2.** Solve the equation

$$\sqrt[3]{(8-x)^2} - \sqrt[3]{(8-x)(27+x)} + \sqrt[3]{(27+x)^2} = 7.$$

**Analysis.** The given equation is an irrational equation. At the same time, a quadratic expression is located under the root sign, and the equation is meaningful for all real numbers [14].

**Abstraction.** Equations of the form

$$\sqrt[3]{(a-x)^2} - \sqrt[3]{(a-x)(b+x)} + \sqrt[3]{(b+x)^2} = c$$

can be reduced to solving a system of equations by means of the substitutions

$$t = \sqrt[3]{(a-x)}, \qquad z = \sqrt[3]{(b+x)}$$

It is known that in the equation

$$\sqrt[3]{(8-x)^2} - \sqrt[3]{(8-x)(27+x)} + \sqrt[3]{(27+x)^2} = 7$$

we introduce the designations

$$t = \sqrt[3]{(8-x)}, \qquad z = \sqrt[3]{(27+x)}.$$

Then, since

$t^3 = 8 - x$ and $z^3 = 27 + x$

we have

$$t^3 + z^3 = 35$$

and

$$t^2 - tz + tz^2 = 7.$$

Multiplying both sides of the equality $t^2 - tz + tz^2 = 7$ by $(t+z)$, we obtain

$$(t+z)(t^2 - tz + z^2) = 7(t+z).$$

Therefore, it can be seen that

$$t + z = 5.$$

As a result, we come to solving the following system of equations:

$$\begin{cases} t + z = 5 \\ t^2 - tz + z^2 = 7 \end{cases} \Rightarrow \begin{cases} z = 5 - t \\ t^2 - t(5-t) + (5-t)^2 = 7 \end{cases} \Rightarrow$$

$$\begin{cases} z = 5 - t \\ t^2 - t(5-t) + (5-t)^2 = 7 \end{cases} \Rightarrow \Rightarrow \begin{cases} z = 5 - t \\ t^2 - 5t + 6 = 0 \end{cases}$$

Applying Vieta's theorem to the second equation of the system, we obtain $t = 2$ and $t = 3$, and respectively $z = 3$ and $z = 2$.

**Generalisation.** By substituting the obtained values of the roots, we find

$$\sqrt[3]{8-x} = 2 \Rightarrow 8 - x = 8 \Rightarrow x = 0$$

and

$$\sqrt[3]{8-x} = 3 \Rightarrow 8 - x = 27 \Rightarrow x = -19.$$

Thus, the solutions are

$$x = 0, \qquad x = -19.$$

**Example 3.** Solve the equation

$$x + \sqrt{1 - x^2} = \sqrt{2}\,(2x^2 - 1).$$

**Solution.** First, we find the domain of definition of the equation:

$$1 - x^2 \geq 0 \Rightarrow (1 - x)(1 + x) \geq 0 \Rightarrow -1 \leq x \leq 1.$$

Consequently, the solutions of the equation must lie in the interval $[-1,1]$. Therefore, it is appropriate to introduce the following substitution [15]:

$$x = \sin t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}.$$

As a result, the above irrational equation is transformed into the following trigonometrik equation:

$$\sin t + \sqrt{1 - \sin^2 t} = \sqrt{2}(2\sin^2 t - 1),$$

that is,

$$\sin t + \cos t = -\sqrt{2}\cos 2t.$$

It is known that

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$$

Thus, we obtain the equation

$$\sin t + \cos t = -\sqrt{2}(\cos t - \sin t)(\cos t + \sin t).$$

Taking common factors from the obtained equation, we come to

$$(\cos t + \sin t)\left[\sqrt{2}(\cos t - \sin t) + 1\right] = 0.$$

Therefore, we come to solving the following two equations:

$$\frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{2}}\sin t = 0$$

and

$$\frac{1}{\sqrt{2}}\cos t - \frac{1}{\sqrt{2}}\sin t = -\frac{1}{2}.$$

The obtained equations can be written as

$$\sin\left(\frac{\pi}{4} + t\right) = 0$$

and

$$\sin\left(\frac{\pi}{4} - t\right) = -\frac{1}{2}.$$

For $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$, their solutions are

$$t = -\frac{\pi}{4}$$

and

$$t = \frac{5\pi}{12}.$$

Since $x = \sin t$, we find the corresponding values of the sine:

$$x_1 = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},$$

and

$$x_2 = \sin\frac{5\pi}{12} = \sqrt{\frac{1 - \cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}.$$

## 4. Conclusion

The study successfully demonstrates that the purposeful integration of general logical activity methods—specifically analysis, synthesis, comparison, generalization, and abstraction—significantly enhances students' problem-solving efficiency and cognitive dexterity when tackling complex irrational equations. The core findings reveal that explicitly structuring mathematical solutions around these logical frameworks moves instruction away from rote, spontaneous learning toward a conscious, systematic development of mathematical competencies. The pedagogical implications of this research are profound; they underscore the necessity of shifting general secondary education curricula toward a "mathematics for the student" paradigm. By teaching students how to dissect, abstract, and generalize algebraic structures, educators can cultivate higher-order logical thinking skills that are directly applicable to real-life problem-solving and international literacy frameworks like PISA and TIMSS. Building upon these insights, further research should explore the longitudinal impact of these explicit logical frameworks on student performance across other advanced mathematical domains, such as calculus or geometry, and investigate the development of specialized digital learning technologies designed to systematically embed general logical methods into remote and hybrid learning environments.

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